A Quantitative Case Study on the Impact of Transaction Cost in High-Frequency Trading Models

High-frequency trading is aimed at systematic earning of a tiny positive alpha on every trade. Since the alpha is tiny, the transaction cost decides the effectiveness of the algorithm.

Executive Summary

High-frequency trading (HFT) strategies, or even arbitrage for that matter, require a quantitative model that runs upon the live stream of various parameters such as price, volume, volatility, etc. and ensures a positive alpha at the closure of a trade. Since the magnitude of this alpha is very low, the transaction costs (often quoted as commissions) play a vital role in the success of the strategy. Often a quant model that works on paper during back-testing is invalidated by the transaction costs. This paper presents a case study for a composite quantitative model to understand how cost affects the HFT algorithm.

Dynamics of the Trading Strategy: The Financial Aspect

Among the various HFT trading models, spread trading is very popular. Below we formulate our own mathematical model to understand and quantify the stability of the spread model.

Background: In a simplified example, if we look at the intraday price (tick data) curve of E-Mini S&P 500 1-month futures versus that of E-Mini S&P 500 2-month futures, then a pattern would emerge where one of the instrument's price would rise whereas the price of the other would fall, or the price of the two tickers might move in the same direction but at different accelerations.

Since the two tickers have the same underlying terms and conditions excluding the maturity date, their price movement in different directions will affect the stability/equilibrium of the system. Hence a mean reversal is bound to happen. In other words, if the E-Mini S&P 500 1-month futures has gone up and the E-Mini S&P 500 2-month futures has declined, then after a while the 2-month futures will start rising and/or the 1-month futures will come down.

Opportunity: Long the ticker that is underpriced and short the one that is overpriced. As we mentioned above, mean reversal is bound to happen. Close this trade once the profit target is achieved and/or when you get another signal in the opposite direction.

Challenges: What do we mean by underpriced/overpriced? We can't really expect the absolute price of the two instruments to cross each other. So how do we identify the timing of this trade?

Second, how do we optimize the transaction costs, since they will have a major impact on the profit target?

Proposed Quantitative Model

As explained in the previous section, the basis of this strategy is that the price of two futures instruments with similar underlying contract terms and conditions but different expiry dates/currency/markets must be highly correlated. Hence, in high-frequency price movement, ticks in the opposite direction cannot last forever and mean reversal is bound to happen.

Since this trading strategy is purely quantitative (does not involve economics/finance, nor is it based on instrument profile), it can be applied to any two futures within the same market or even across markets.

To maximize profit, instruments with large tick size and lower commissions must be chosen. A good example would be the Shanghai Index Futures with a tick size of 0.2 and transaction cost of 0.08 – i.e., less than half of the tick size.

While acquiring the position (or closing the trade), the slippage/liquidity risk is the only real risk that this strategy carries. Since this is a dollar-neutral strategy, the other major market risk factors (delta, gamma, vega and theta) are nullified to a great extent.

Notations/Assumptions Used in the Model

- We posit a simple model of HFT spread for two futures with same underlying terms and conditions and in the same market. For example, E-Mini S&P 500 1-month (hereon referred to as X) versus E-Mini S&P 500 2-month (hereon referred to as Y).
- The prices at any given interval for X will be denoted as Pxi (Pbx1, Pbx2, Pbx3 ... Pbxn for bid, Pax1, Pax2, Pax3 ... Paxn for ask).
- The prices at any given interval for Y will be denoted as Pyi (Pby1, Pby2, Pby3 ... Pbyn for bid, Pay1, Pay2, Pay3 ... Payn for ask).
- The respective volume figures for Pxi prices will be denoted as Vxi (Vbx1, Vbx2, Vbx3 ... Vbxn for bid, Vax1, Vax2, Vax3 ... Vaxn for ask).
- The respective volume figures for Pyi prices will be denoted as Vyi (Vby1, Vby2, Vby3 ... Vbyn for bid, Vay1, Vay2, Vay3 ... Vayn for ask).
- We assume a timer to periodically (500 milliseconds for example) check the latest price and volume for both the instruments. Thus, we need a sampling mechanism to synchronize the latest prices at any given time-stamp.

We assume that the netting of the trading account will happen at the exchange level automatically. For exchanges where netting doesn't happen by default, a separate netting mechanism will have to be implemented.

We assume a separate time-out mechanism attached with every opened trade. Since this is an HFT trade, a position cannot be kept open for long periods. We apply a simple rule of a time-out at double the average time needed to close 90% of trades. When the time-out occurs, the trade is force-closed at market price irrespective of whether a profit or loss is booked.

Creating the Basic Model

- Let's assume a virtual instrument Z where the bid price of Z would be Pzb = bid of Y – ask of X.
- Ask price of Z would be Pza = ask of Y – bid of X.
- Now, compute the tick prices (bid and ask) of Z for the last one minute (this time can be configured to back-test and optimize).
- Derive the moving average of bid price of Z (Zbma):
  - Zbma = SUM{(Pby1-Pax1) + (Pby2-Pax2) + (Pby3-Pax3) + ... + (Pbyn-Paxn)} / n (n is the last 60th second price sample collected)
- Derive the moving average of ask price of Z (Zama):
  - Zama = SUM{(Pay1-Pbx1) + (Pay2-Pbx2) + (Pay3-Pbx3) + ... + (Payn-Pbxn)} / n
- **Trade Entry Criteria**:
  - When Pzb (current bid price of Z) > Zama +K: hit the bid of Z (short Y at the price of Pby, long X at the price of Pax), volume of X and volume of Y would be same = Min (Vby ,Vax) – Entry_Criteria-1
  - When Pza (current ask price of Z) < Zbma-K: hit the ask of Z (long Y at the price of pay, short X at the price of Pbx), volume of X and volume of Y would be same = Min (Vay ,Vbx) – Entry_Criteria-2

- **What is K?** For now, assume K to be a threshold – let's say the profit target. Later, we will investigate the value of K while measuring the transaction cost.

- **Trade Exit Criteria**:
  - While we are in the trade, we need to keep checking the prices of both X and Y, and keep measuring Pzb and Pza.
  - If we had opened our trade by hitting the bid of Z, let's say at a price of Pzbi (Pbyi – Paxi), we need an ask price of Pzaj (Payj – Pbxj) that would be <Pzbi – K. So, hit the ask of Z (long Y at the price of pay, short X at the price of Pbx) when Pzaj < Pzbi – K {The volume of X and volume of Y would be same = Min (Vay ,Vbx).}
  - If we had opened our trade by hitting the ask of Z, let's say at a price of Pzai (Payi – Pbxi), we need a bid price of Pzbj (Pbyj – Paxj) that would be >Pzbi + K. So, hit the bid of Z (short Y at the price of Pby, long X at the price of Pax) when Pzbj > Pzai + K {The volume of X and volume of Y would be same = Min (Vby, Vax).}

Incorporating Transaction Cost

It is important to ensure that we are acquiring the exact volume we are asking for – hence we need to choose the order types accordingly. Also, it is very important to make sure that we have the exact volume in both the instruments.

The profit target K should also incorporate the transaction costs.

- Now let's take the Entry_Criteria-1
  - Pzb (current bid price of Z) > Zama + K
  - Or, Pzb – Zama > K

Let's say the transaction cost (brokerage + exchange execution cost) for X = Tx, and that of Y = Ty. Also let's assume that the tick size for instrument X = tx and that of instrument Y is ty.

- In that case, the total execution cost for a trade entry + exit would be Tran(x + y) = 2*(Tx + Ty).
- If we assume a profit of at least one tick per instrument per trade then the total profit target would be Profit(x + y) = (tx + ty).
- Hence the value of K becomes – K = 2*(Tx + Ty) + (tx + ty): EQ-1
- Now, let's further express the transaction cost in terms of the number of ticks. For example, Tx = Cx*tx (transaction cost for X instrument is Cx times the tick size of X) and Ty = Cy*ty (transaction cost for Y instrument is Cy times the tick size of Y).
- Then ***EQ-1*** becomes:
  - K = 2*(Tx + Ty) + (tx + ty)
  - Or K = 2*(Cxtx + Cyty) + (tx + ty)
  - Or K = tx (2*Cx + 1) + ty (2*Cy + 1): EQ-2

From EQ-2 we clearly see that value of K is directly proportional to Cx and Cy, which means that if the transaction cost is lower, K will be lower as well.

If we statistically analyze the data for any spread trading instrument couple, then we will see that the value of Pzb – Zama will never be very high. Typically, the value of K never goes beyond 10 ticks.

- Based on the above statement let's modify EQ-2 assuming tx = ty = t and Cx = Cy = C
  - K = tx (2*Cx + 1) + ty (2*Cy + 1)
  - Or 10t = t(2*C + 1) + t(2*C + 1)
  - Or 10t = 2t + 4tC
  - Or 4tC = 8t
  - Or C = 2

From the above equation, we can see that if this HFT trading strategy is to become feasible, the maximum possible transaction cost is two ticks.

For Commodities: Further Enhancement for Cross-Market Trading

- The most popular spread trading concept is relevant to the commodities market where, for example, 6-month futures contracts for gold on CME and MCX, or CME versus Shanghai, would typically have very similar terms and conditions.
- However, the challenge will be in bringing either of the instrument prices in the equilibrium range of the other. For example, let's say the price of a CME Gold 6-month futures contract is in the $4,000 range, whereas the MCX contract is in the range of 80,000 INR.

There are two factors that must be factored in before beginning the trading:

- The lot size of the two contracts might be different and will need to be determined from the individual contract specifications. Let us say gold 6-month futures CME (instrument X in our equation) has a lot size of Lx, and the same for MCX (instrument Y in our equation) is Ly.
- The forex rate; decide on a reference rate for FX conversion at the beginning of the day. Let's say USDINR conversion rate = Ul.

Now, we have to create a constant factor D that will bring either of the instrument prices in the range of the other. In our example we will bring the MCX prices (Y) in the range of CME prices.

- So, the value of D would be: D = Lx /(Ly*Ul)

Henceforth, we will keep on multiplying all the MCX prices (Pby and Pay) with the factor D before performing further calculations.

While executing the trades, we need to understand that we opened our trade by looking at the converted price of MCX; however, our intention is mainly to estimate the notional price during execution. Let's say we opened a trade when the actual MCX price was Pby/Pay and volume Vby/Vay respectively. We decided to open the trade by looking at a price Pby*D / Pay*D.

Our ambition here is to buy, let's say, Vax lots. Then that also means we want to acquire Vax*Pby*D amount of the position. Hence, the lot size we will execute in MCX is = Vax*D, Price Pby.

There is another challenge in cross-market trading – guaranteeing the volume. For trading within a single market, normally we have exchange-supported conjugate order types to place where we will definitely get the ordered volume (or else the trade will not execute). There would be no scenario where we end up picking unwanted volume for X and Y. However, in a cross-market scenario, two instruments are being traded in two different exchanges. Hence, there are no exchange-supported order types to guarantee the volume simultaneously for both the instruments. This challenge needs to be handled technically.

Conclusion

As we see from the simplification of EQ-2, even if we assume the real-time spread between two futures of the same instrument to be as high as 10 ticks (a rare case), the transaction cost has to be as low as two ticks for the trading strategy to work. Very selective instruments (e.g., Shanghai Index Futures) have a higher magnitude of ticks that might make the strategy feasible. Otherwise, the quantitative model fails on high transaction cost.

However, the cross-market spread for commodities might work well for this case. The correlation between two gold futures instruments, for example, of two different markets should be high enough for this strategy to work, though the practically achievable spread should be higher.

(*NOTE: This paper does not claim any accuracy/profitability for the described quantitative model for any instrument. The quantitative strategy described above is well known to the HFT algo traders, and we do not claim it is a unique model.*)

About the Author

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